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A NONLINEAR DISPERSIVE WAVE MODEL FOR IRREGULAR BOUNDARIES

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The wave model adopted in this work is the single-component form of the fully-dispersive weakly-nonlinear wave equations of Nadaoka *et. al.* (1997). The wave model is valid for arbitrary depths, ranging from infinitely deep to very shallow waters. The recently proposed nonlinear wave model is re-expressed in boundary fitted non-orthogonal curvilinear co-ordinate system for simulating wave motions in domains with irregular boundaries. The co-ordinate transformation converts an irregular physical domain into a rectangular computational domain, which allows for accurate numerical computations using finite-differences approximations. Thus, the boundary conditions for irregular vertical enclosures surrounding a typical physical domain, such as a port or harbour, are satisfied accurately. The transformed continuity and momentum equations are discretized by finite difference approximations using non-staggered grids, where the free surface displacement, the velocity components, and the grid metrics are defined at the grid intersections. The second-order central-difference formulae are used for approximating all the partial derivatives both in time and space. The basic algorithm is divided into two stages: In the first stage, the velocity components are computed by solving the momentum equations until a specific convergence criterion is met. The solution of the velocity components requires the use of Thomas algorithm, which is quite efficient in the solution of tridiagonal matrix systems. Then, using the final velocity components, the free surface elevation is obtained from the continuity equation at each time step. The boundary conditions on the free surface and on the bottom are automatically satisfied by the wave equations. It then remains to specify the conditions on the incident and outgoing boundary and on the vertical enclosures surrounding the domain. The conditions at the incoming boundary is easily specified by introducing a specified incident wave field; likewise, the wall condition, which states that the velocity normal to the wall surface must vanish, is satisfied easily in the transformed rectangular computational domain. The radiation condition usually presents difficulties since there is no perfect radiation condition for nonlinear directional waves leaving the domain. Here, the second-order radiation condition of Engquist and Majda (1977) is used to minimise the artificially reflected waves from the outgoing boundary. A sample computation using the above scheme for the case of wave propagation over a topographical lens (Whalin, 1972) is performed and show good agreement with measurements. Comparisons of computational results with experimental measurements of Nadaoka *et. al.* (1994) of horizontal and vertical orbital velocities of nonlinear unidirectional waves propagating over a submarine bar show good agreement and prove the superiority of the new wave model over an improved Boussinesq model. These comparisons are taken to be indicative of the same order of accuracy for orbital velocity predictions of directional wave motions. This in turn results in better estimation of wave motions and forces. The numerical model is applied to a sample case for simulating waves inside a harbour. Results obtained appear quite realistic and give confidence for practical applications of the model introduced.