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APPLICATIONS OF A DEPTH INTEGRATED WAVE MODEL

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ABSTRACT

Various different applications of a depth integrated nonlinear dispersive wave model are presented. The applications cover a wide range of areas, such as wave transformations in the coastal regions and harbors, computation of linear and nonlinear wave forces acting upon bottom mounted surface piercing piles, and tsunami simulations for a given underwater earthquake scenario. In all these applications relatively low computational cost of depth-integrated models is emphasized.

KEY WORDS: Depth-integrated wave model; shoaling; refraction; diffraction; reflection; wave forces; tsunami simulations.

WAVE MODEL AND NUMERICAL CODE

The wave model used in this work is due to Nadaoka et. al. (1997)

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}\left[\left(\frac{C_p^2}{g} + \zeta\right)u\right] + \frac{\partial}{\partial y}\left[\left(\frac{C_p^2}{g} + \zeta\right)v\right] = 0$$

$$r\frac{\partial u}{\partial t} + g\frac{\partial \zeta}{\partial x} + \left(1 - 3\frac{\omega^2 C_p^2}{g^2}\right)\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)$$

$$-\frac{1}{\omega^2 C_p^2}\frac{\partial^2}{\partial x \partial t}\left[C_p^4(1-r)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] = 0$$

$$r\frac{\partial v}{\partial t} + g\frac{\partial \zeta}{\partial y} + \left(1 - 3\frac{\omega^2 C_p^2}{g^2}\right)\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right)$$

$$-\frac{1}{\omega^2 C_p^2}\frac{\partial^2}{\partial y \partial t}\left[C_p^4(1-r)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] = 0$$

where ζ is the free surface deformation, u and v are the depth-averaged velocities in the x- and y-directions, respectively. $r=C_g/C_p$ is the ration of the group velocity C_g to the phase velocity C_p, as given by linear theory for a given circular frequency ω and local water depth h.

For numerical solutions the wave equations may be discretized by central difference approximations using Arakawa-C grid system. Usually such a discretization approach works well; however, for very long waves the ratio $r\rightarrow 0$ hence the scheme breaks down. In order to overcome this defect a different approach, as formulated by O'Brien and Hurlburt (1972) for two-layer shallow water equations, is adopted. Accordingly, the continuity equation is discretized in time first, then differentiated with respect to x and then substituted into the x-momentum equation. Subsequently, this re-arranged momentum equation is discretized again. For the y-momentum equation the same procedure is repeated. The resulting numerical scheme works for the special case of $r\rightarrow 0$ as well, without any numerical stability problem. All the simulations shown in this work are obtained from a FORTRAN program constructed in accordance with the discretization scheme just described.

An equally important aspect of the FORTRAN program is the automated recognition of lands and open boundaries and the use of the corresponding boundary conditions. The program generates the grids and identifies the type of boundaries when the water depths are given by negative values and the lands by zeros. All the boundaries with negative water depths are treated as open boundaries while zero depths are recognized as lands and treated as fully-reflecting wall boundaries. The waves are always generated inside the computational domain and symmetrically radiated away from the generation line. Since the waves are not generated along a boundary line, the waves reflected from within the domain do not cause any undesirable re-reflection problems.

Time-dependent real time simulations of linear or nonlinear waves provide 3-D perspective views of the entire wave field for the duration of the simulation. After the wave field develops completely the steady-state solution can be obtained by computing the wave heights at every grid point in the domain. Afterwards a 3-D view of the steady-state solution can be constructed as shown for some test case applications.

APPLICATIONS OF NUMERICAL MODEL

Various different applications of the numerical model are now given. The first case is wave scattering behind a gap. Then, cnoidal wave force computations for a cylindrical pile are considered and finally a simulation of waves due to an underwater earthquake in the Sea of Marmara is presented.

Wave Scattering Behind a Gap

Wave scattering behind a gap of two wavelength width is considered. A 3-D perspective view of the time-dependent simulation after 30 periods have elapsed from the cold start is shown in Fig. 1.

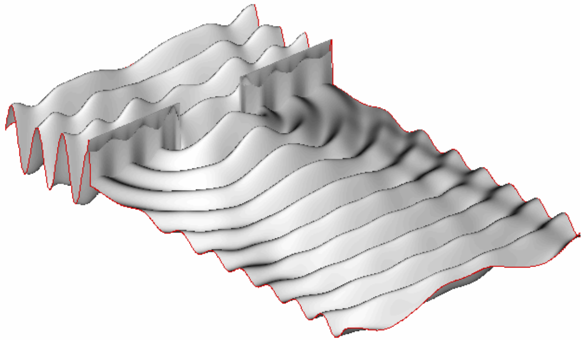

Fig. 1: 3-D view of wave scattering behind a gap at 30 periods.

Johnson’s (1952) solution as given in Shore Protection Manuel (1984) is given in Fig. 2a below for comparison with the numerical solution shown in Fig. 2b. The agreement between the two solutions seems satisfactory.

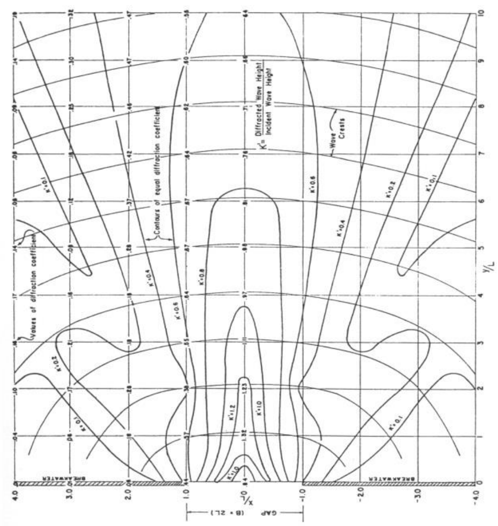

Fig. 2a: Johnson’s (1952) diagram of scattering behind gap.

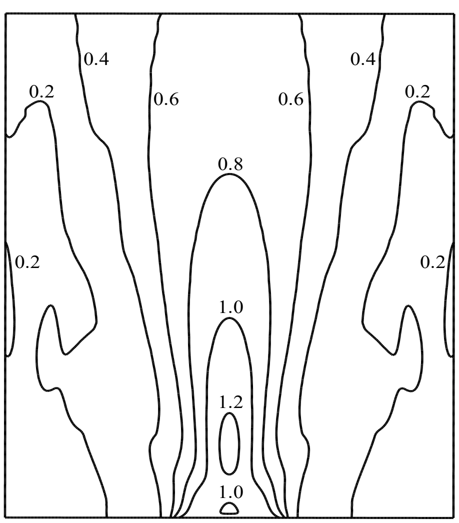

Fig. 2b: Diagram constructed from the numerical simulation.

Cnoidal Wave Forces on a Pile

A perspective view of cnoidal waves in presence of a circular cylindrical pile is shown in Fig. 3a.

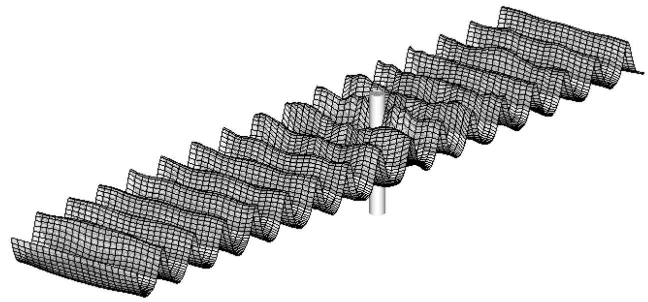

Fig. 3a: Cnoidal waves impinging upon a circular cylindrical pile.

The time history of the total force acting on the pile due to the cnoidal waves is shown in Fig. 3b.

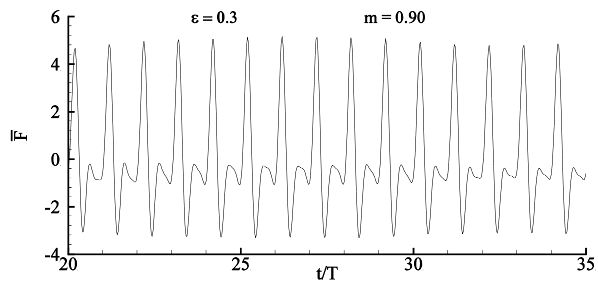

Fig. 3b: Time history of force acting on a pile due to cnoidal waves.

Waves Generated by Underwater Earthquake in the Sea of Marmara

3-D view of the initial generation of waves due to an underwater earthquake in a region of the Sea of Marmara is shown in Fig. 4.

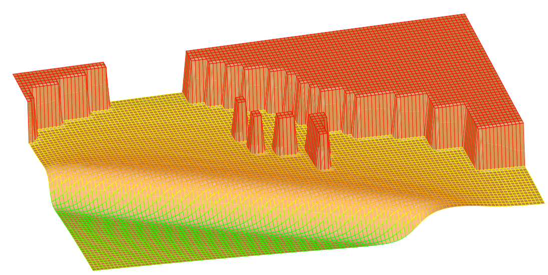

Fig. 4: 3-D view of wave generation due to underwater earthquake.

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